Master-Prüfung

“Allgemeines Gleichgewicht und Social Choice:

Ökonomik und Ethik”

SS 2023

6 ECTS

Duration: 90 Minutes

August 9, 2023

Prof. Lutz Arnold

Please write legibly:
Surname:
Given name:
Matr.-nr.:

To be filled out by examiner:

1	2	3	4	5	6	7	$\sum$

Solve five out of the seven Problems!

You can obtain a maximum of **20 points** in each Problem.

Always make use of given numbers from the start and file your solutions onto the examination sheet.

Symbols not explicitly defined in the Problems have to be adopted from the lecture notes.

Please check before the examination time starts that your exam sheet contains all pages. It should start with page 1 and end with page 17.

A non-programmable calculator is allowed to be used.

Problem 1: Sorites paradox and transitivity of preferences

(a) Imagine a heap of sand consisting of 1,000,000 grains. State two sensible premises about what a heap of sand is and under what manipulation it continues to be one.

(b) Applying the two premises from part (a), what can you conclude about 999,999 grains of sand?

(c) Iterating the procedure from part (b), what can you conclude about a single grain of sand? Is this reasonable?

(d) Which three properties does a preference relation R have to satisfy in order to be *rational*? Explain each of them both in words and in terms of binary relations.

(e) You are offered a cup of coffee with $n \in \{0, 1, ..., 1{,}000\}$ grains of sugar in it. Assume you like black coffee. What does this mean for your preference over x_0 versus $x_{1,000}$?

(f) What is your likely preference over x_n versus x_{n+1}? Which property of R from part (d) can be used to derive the implied preference over x_0 versus $x_{1,000}$? Explain. Is this generally consistent with rationality?

(g) Which weakening of the critical condition in part (d) makes the inconsistency in part (f) vanish? Argue that there is no problem with rationality with the weakened condition.

Problem 2: Edgeworth box and production possibilities frontier

(a) State the budget constraint for the Walrasian economy with $L+1$ good where individuals i owns fractions $\bar{\theta}_{ijl}$ of the firms.

(b) Assume that there are two goods ($L = 1$) and one (price taking) firm producing good 1 ($J_1 = 1$). State the utility maximization problem of individual i. Let the budget constraint hold with equality and assume all relevant functions are differentiable in what follows.

(c) Show:

$$\frac{\frac{\partial u_i(\boldsymbol{c_i})}{\partial c_{i0}}}{\frac{\partial u_i(\boldsymbol{c_i})}{\partial c_{i1}}} = \frac{p_0}{p_1} = f'_{11}(k_{11}).$$

(d) Construct the production possibilities frontier. What are the variables plotted along the axes?

(e) Assume there are two individuals ($I = 2$). Add the Edgeworth box to the diagram in part (d).

(f) Add the contact curve to the diagram. Which property do allocations on it satisfy? Explain.

(g) Illustrate a Walrasian equilibrium in the diagram. Mark the consumption levels c_{10}, c_{20}, c_{11}, and c_{21}.

Problem 3: The welfare theorems

(a) State the First welfare theorem for the Walrasian model $\mathscr{W}$.

(b) Suppose there is a feasible allocation $((\boldsymbol{c}'_i)_{i=1}^I, \boldsymbol{k}')$ that is Pareto-superior to the equilibrium allocation. Prove:

$$\sum_{i=1}^{I} \boldsymbol{p}\boldsymbol{c}'_i > \sum_{i=1}^{I} \boldsymbol{p}\boldsymbol{c}_i.$$

Explain the steps in your proof.

(c) Use market clearing, profit maximization, and feasibility to produce a contradiction.

(d) Mark three mistakes in the following statement of the Second welfare theorem:

THEOREM (**Second welfare theorem**): *Suppose an equilibrium of* $\mathscr{W}$ *exists for all* $(c_{i0}, \bar{\boldsymbol{\theta}}_i)_{i=1}^I$ $(\in \mathbb{R}_+^{(1+J)I})$*. Then, if* $((\boldsymbol{c}_i^*)_{i=1}^I, \boldsymbol{k}^*)$ *is a Pareto-optimal allocation, there exist* $\boldsymbol{w}$ *with* $\sum_{i=1}^I w_i = 1$ *and* $\boldsymbol{p}$ *such that* $((\boldsymbol{c}_i^*)_{i=1}^I, \boldsymbol{k}, \boldsymbol{p})$ *is an equilibrium of* $\tilde{\mathscr{W}}$*.*

(e) What are the implications of the Second welfare theorem for distributive justice and redistributive policies in market economies?

Problem 4: Ultimatum game

(a) Consider an "Ultimatum game", in which Player 1 proposes an allocation (c_1, c_2) of 20 units of consumption to Players 1 and 2. Player 2 accepts the allocation or not. If she rejects it, both players obtain nothing. Write down the game in extensive form. What is the "budget constraint"?

(b) Describe the subgame perfect equilibrium with regular preferences $u_i(c_i) = c_i$.

(c) Given Fehr-Schmidt-preferences and linear consumption utility $u_i(c_i) = c_i$, what is Player 2's utility function? Which condition characterizes offers that she will accept? Explain the threshold value carefully.

(d) Distinguish the three cases, $c_1 = c_2$, $c_1 < c_2$, and $c_1 > c_2$. For each case, for which values of c_2 does Player 2 accept Player 1's proposal? Explain each case separately! Which constraint is the strictest one?

(e) Use the "budget constraint" to show that Player 1 has to make an offer with $c_2 \geq 20\frac{\alpha_2}{1+2\alpha_2}$ in order to make Player 2 accept. Describe the subgame perfect equilibrium and the Players' resulting payoffs.

Problem 5: Utilitarianism

(a) Write down the utilitarian social welfare function (SWF).

(b) Do you consider utilitarianism to be a teleological or a deontological ethic? Explain!

(c) Suppose an allocation $x \in X$ maximizes the utilitarian SWF. To prove Pareto-efficiency of x by contradiction, what do you assume about some alternative allocation x'? Explain.

(d) Complete the proof by comparing the values of the SWF resulting from x and x', respectively, and carefully explaining the contradiction!

(e) What characterizes the points of a social indifference curve (SIC)?

(f) For an economy consisting of $I = 2$ individuals, calculate and draw the SIC when employing Utilitarianism. Mark the variables plotted along the axes in your diagram.

(g) What kind of utility function defined over two-dimensional consumption vectors yields individual indifference curves that resemble this SIC? What does that imply for substitutability?

Problem 6: Contraction of decisive sets (single-profile case)

(a) Define *decisiveness* of a subset $H \subset I$ over a pair of alternatives $\{x, y\}$ and overall.

(b) Define *spread of decisiveness.* Give both a formal statement and an intuitive explanation!

(c) Consider a decisive set H with $\#H > 1$. For a partitioning of H into two proper subsets H_1 and H_2, state three properties that these subsets fulfill.

(d) Consider the following profile of (rational) preferences:

$$x \; P_i \; z \; P_i \; y, \;\; i \in H_1$$

$$y \; P_i \; x \; P_i \; z, \;\; i \in H_2$$

$$z \; P_i \; y \; P_i \; x, \;\; i \in I \setminus H.$$

How is this profile called? Define *diversity.*

(e) Given the preferences in the above array, check who (dis-) agrees with whom on what by completing the table below.

x vs. y	x vs. z	y vs. z

(f) What can you infer from the above table for the social ordering R?

(g) Assuming that (socially) $z\,R\,y$, find a decisive proper subset of H. Spell out each single step in your argument.

(h) Assuming that (socially) $\sim(z\,R\,y)$, find a decisive proper subset of H. (Hint: Make use of completeness of R.) Spell out each single step in your argument.

(i) State the Contraction of decisive sets lemma. Prove it using your answers to parts (g) and (h). State and prove the Single-profile Arrow theorem.

Problem 7: Gibbard's oligarchy theorem and Arrow's impossibility theorem

(a) Define *Independence of irrelevant alternatives (IIA)*. Explain this property.

(b) Define: H is *decisive* over $\{x, y\} \subset X$ and H is *decisive*. Explain.

(c) State (without proof) the Spread of decisivesness lemma.

(d) Suppose $\{x, y\}$ and $\{w, z\}$ are disjoint. Consider a preference profile $\mathbf{R} \in \boldsymbol{\mathcal{R}}$ with

$$w \, P_i \, x \, P_i \, y \, P_i \, z, \quad i \in H$$

$$w \, P_i \, x, \; y \, P_i \, z, \quad i \in I \backslash H.$$

Show step by step that $w \, P \, z$.

(e) Define an *oligarchy*. What kind of veto power does oligarchy imply?

(f) State (without proof) Gibbard's oligarchy theorem.

(g) Define a *dictator*.

(h) State (without proof) Arrow's impossibility theorem.

(i) Make some remarks on Arrow's contribution to social choice theory.