Regensburg days on non-archimedean geometry

Regensburg, July 25th to 27th 2023

Organizers: Walter Gubler, Klaus Künnemann, and Enrica Mazzon

Tuesday

9h30 - 10h30	Vladimir Berkovich	Hodge theory for non-Archimedean analytic spaces
11h00 - 12h00	Antoine Ducros	A non-archimedean Chevalley theorem
14h00 - 15h00	Martin Ulirsch	Archimedean and non-Archimedean P=W phenomena
15h30 - 16h30	Ilya Tyomkin	Tropical methods in irreducibility problems

Wednesday

9h30 - 10h30	Andreas Mihatsch	Intersection on Drinfeld 4-Space
11h00 - 12h00	Joe Rabinoff	Real-valued differential forms on Berkovich spaces via harmonic tropicalizations
14h00 - 15h00	Vlerë Mehmeti	Variation of Hausdorff dimensions of limit sets
15h30 - 16h30	Omid Amini	The tropical Hodge conjecture
19h00		Conference Dinner *Bischofshof am Dom*

Thursday

9h30 - 10h30	Michael Temkin	The different function on Berkovich spaces
11h00 - 12h00	Léonard Pille-Schneider	Continuity of families of Monge-Ampère measures

Place: All lectures take place in lecture room M311 in the math department of the University of Regensburg.

Conference dinner: The conference dinner takes place at *Bischofshof am Dom* which is situated on the northern side of the cathedral.

`https://www.hotel-bischofshof.de/?lang=en`

Conference website: Please check the conference website

`sfb-higher-invariants.app.uni-regensburg.de/index.php?title=Regensburg_days_on_non-archimedean_geometry`

for more information and updates of this program.

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Abstracts

Omid Amini: The tropical Hodge conjecture

I will present the formulation of the Hodge conjecture for tropical varieties and explain a proof in the case the tropical variety is triangulable over the filed of rational numbers. This provides a partial answer to a question of Kontsevitch.
The proof uses Hodge theoretic properties for tropical varieties, Hard Lefschetz, Hodge-Riemann bilinear relations and weight-monodromy conjecture, established in companion work, that will be reviewed in the talk. The Hodge theoretic results generalize to the global setting the work of Adiprasito-Huh-Katz in combinatorial Hodge theory, by going in the local setting beyond the case of matroids and their Bergman fans, and provide answers to conjectures of Mikhalkin and Zharkov.
The talk is based on joint work with Matthieu Piquerez.

Vladimir Berkovich: Hodge theory for non-Archimedean analytic spaces

By Deligne's Hodge theory, the integral cohomology groups $H^n(\mathcal{X}^h, \mathbf{Z})$ of the $\mathbf{C}$-analytification of a separated scheme $\mathcal{X}$ of finite type over $\mathbf{C}$ are provided with a mixed Hodge structure, functorial in $\mathcal{X}$. Given a non-Archimedean field K isomorphic to the field of Laurent power series $\mathbf{C}((z))$, there is a functor $\mathcal{X} \mapsto \mathcal{X}_K^{\mathrm{an}}$ that takes $\mathcal{X}$ to the K-analytification of $\mathcal{X}_K = \mathcal{X} \otimes_{\mathbf{C}} K$. I'll describe a Hodge theory for the category of strictly K-analytic spaces with the property that each compact analytic subdomain is isomorphic to an analytic domain in a boundaryless space. It extends Deligne's Hodge theory through the above functor and generalizes previously known complex analytic constructions.

Antoine Ducros: A non-archimedean Chevalley theorem

Let $f\colon Y \to X$ be a map between compact analytic spaces. The structure of the image $f(Y)$ is well-known when f is proper (this is a Zariski-closed subset of X) or flat (this is a compact analytic domain of X), but far less understood in general. In this talk I will present what I see as kind of a weak Chevalley theorem. It states essentially that $f(Y)$ has a nice decomposition into finite many elementary pieces, each of which is a Zariski-closed subset of an analytic domain of X (neither open nor closed in general). The proof relies upon my former work on non-archimedean flattening techniques (à la Raynaud-Gruson) and on étale descent.
If I have time, I will also explain how these results imply that flatness can be detected naively if one uses Huber's local rings instead of Berkovich's.

Vlerë Mehmeti: Variation of Hausdorff dimensions of limit sets

The limit set of a discrete group of Möbius transformations acting on the projective line is the set of limit points of an orbit, and is a fractal. Over $\mathbb{C}$, it has been extensively studied since the 19th century.
I will be speaking of the Hausdorff dimension of limit sets over the Berkovich projective line, obtained by developing a theory of Patterson-Sullivan measures in this non-Archimedean setting. In the particular case of Schottky groups, we will see an algorithm for its computation. In 2021, Poineau and Turchetti constructed a moduli space for Schottky groups over Berkovich spaces

over $\mathbb{Z}$. I will present a result on the continuity of the Hausdorff dimension of the limit sets over said moduli space. This is based on ongoing joint work with Nguyen-Bac Dang.

Andreas Mihatsch: Intersection on Drinfeld 4-Space

Drinfeld's half space of dimension n is a certain formal scheme that parametrizes p-divisible groups with additional multiplication by a central division algebra of invariant $1/n$. It arises naturally during the p-adic uniformization of certain unitary Shimura varieties. Motivated by questions related to cycles on these varieties, I will formulate an intersection problem for Drinfeld's half space and present a conjectural expression for the arising intersection numbers. I will then discuss results in dimension 4. This is joint work with Qirui Li.

Léonard Pille-Schneider: Continuity of families of Monge-Ampère measures

Let R be a discrete valuation ring, and $(X, L)/R$ a normal integral polarized scheme. We prove that if ϕ is a continuous family of plurisubharmonic metrics on the Berkovich analytification X^{an}, varying in a psh (which here means convex) way with respect to the base Spec R, then the associated family of fiberwise Monge-Ampère measures is weakly continuous on the total space. I will also try to discuss some applications.

Joe Rabinoff: Real-valued differential forms on Berkovich spaces via harmonic tropicalizations

I will report on ongoing work with Gubler where we develop a variant of Chamber-Loir–Ducros' framework for studying smooth differential forms on Berkovich spaces, in which harmonic functions are by definition smooth. The first technical difficulty is in finding an a priori definition of what "harmonic" means, which involves numerical triviality of line bundles on Temkin's reductions of germs. With this in place, we define a harmonic tropicalization to mean a moment map whose coordinates are harmonic functions; the second difficulty is then to prove that harmonic tropicalizations satisfy the balancing condition. I will discuss these and other aspects of the theory.

Michael Temkin: The different function on Berkovich spaces

I'll tell about my work in progress with Katharina Hübner about properties of the different function associated with a quasi-finite morphism $f\colon Y \to X$ of Berkovich spaces. In particular we will see that if the ground field is defectless, then this real valued function on Y is pathwise continuous and integral PL on skeletons. The situation for an arbitrary ground field is less clear, but I will report about an example in which Y is a curve and the different is PL but has a corner at a point of type 3. To the best of our knowledge this is a first example of a natural geometric function discovering such a peculiar behaviour.

Ilya Tyomkin: Tropical methods in irreducibility problems

In my talk, I will discuss a tropical approach to classical irreducibility problems in Algebraic Geometry. I will explain how to prove the irreducibility of Moduli spaces of curves, Severi varieties, and Hurwitz schemes in arbitrary characteristic by investigating the properties of tropicalizations of families of curves. The talk is based on a series of joint works with Karl Christ and Xiang He.

Martin Ulirsch: Archimedean and non-Archimedean P=W phenomena

Let X be a smooth projective complex variety. Simpson's non-abelian Hodge correspondence provides us with a real analytic isomorphism between the Betti moduli space of characters of

$\pi_1(X)$ and the moduli space of topologically trivial semistable Higgs bundles on X. The P=W conjecture, recently proved by Maulik–Shen and Hausel–Mellit–Minets–Schiffmann, predicts that the perverse filtration on the cohomology of the Dolbeault moduli space agrees (up to index shift) with the weight filtration on the cohomology of the Betti moduli space, when X is a compact Riemann surface. In this talk I will report on a project, in which we extend this $P = W$ phenomenon to X being a complex abelian variety, where it takes a particularly simple form. The insights gained in this situation lead us to a non-Archimedean incarnation of the $P = W$ phenomenon on the ℓ-adic cohomology of the Betti/Dolbeault moduli space of an abelian variety X over an algebraically closed non-Archimedean field K of characteristic zero with maximally degenerate reduction. A central new insight is that the reduced cohomology of the tropicalization of the moduli space of topologically trivial vector bundles on X plays a role as a correction term.