Workshop
"Regensburg days on non-archimedean geometry"

September 30 – October 2, 2019

Organized by: Walter Gubler and Klaus Künnemann

All lectures are held in Room M 311

Monday, September 30

09:00 - 10:00	Antoine Ducros (Paris)	Flattening in Berkovich geometry
10:00 - 10:30	*Coffee break*	
10:30 - 11:30	Sam Payne (Austin)	Topology of moduli spaces of tropical curves
11:45 - 12:45	Dhruv Ranganathan (Cambridge)	Three stories about curves in genus one
12:45 - 14:00	*Lunch time*	
14:00 - 15:00	Annette Werner (Frankfurt)	Degenerations of vector bundles and Mustafin varieties
15:00 - 15:30	*Coffee break*	
15:30 - 16:30	José Burgos Gil (Madrid)	Convex analysis on polyhedral spaces

Tuesday, October 1

09:00 - 10:00	Michael Temkin (Jerusalem)	Reduction and lifting problems on Berkovich curves
10:00 - 10:30	*Coffee break*	
10:30 - 11:30	Johannes Nicaise (London)	Motivic specialization and rationality problems
11:45 - 12:45	Kristin Shaw (Oslo)	
12:45 - 14:00	*Lunch time*	
14:00 - 15:00	Tony Yue Yu (Paris)	The Frobenius structure theorem for affine log Calabi-Yau varieties containing a torus
15:00 - 15:30	*Coffee break*	
15:30 - 16:30	Martin Ulirsch (Frankfurt)	Tropicalizing the moduli space of curves with level structure
16:45 - 17:45	Johannes Rau (Tübingen)	The dimension of amoebas
19:30	*Conference dinner "Leerer Beutel"*	

Wednesday, October 2

09:00 - 10:00	Vladimir Berkovich (Rehovot)	Comparison theorem for de Rham cohomology of non-Archimedean analytic spaces
10:00 - 10:30	*Coffee break*	
10:30 - 11:30	Ilya Tyomkin (Beer Sheva)	Irreducibility problem for Severi varieties
11:45 - 12:45	Hannah Markwig (Tübingen)	Modifications, faithful tropicalization and moduli spaces of plane tropical curves
12:45 - 14:00	*Lunch time*	
14:00 - 15:00	Joseph Rabinoff (Atlanta)	Abelian schemes are log-Néron models
15:00 - 15:30	*Coffee break*	
15:30 - 16:30	Kazuhiko Yamaki (Kyoto)	Ample divisors on tropical varieties

Abstracts

Vladimir Berkovich: Comparison theorem for de Rham cohomology of non-Archimedean analytic spaces

In a work in progress, I defined integral "etale cohomology" groups for certain non-Archimedean analytic spaces over the field of Laurent power series with complex coefficients. They are finitely generated abelian groups that give rise to the l-adic etale cohomology groups and the de Rham cohomology groups of such a space X. The latter comparison isomorphism depends on the choice of a uniformizing element of the power series ring or, equivalently, on the choice of a universal covering of the punctured complex plane. One of the reasons for this is that the construction of the integral etale cohomology groups depends on such a choice. In this talk Ill give a description of the comparison result in the form an isomorphism between two local systems on a "classifying space" of universal coverings of the punctured complex plane. The first one is a complexified local system of the integral etale cohomology groups of X, and the second one is the local system of horizontal sections of a vector bundle with an integrable connection associated to the de Rham cohomology groups of X.

Jose Ignacio Burgos Gil: Convex analysis on polyhedral spaces

Convex functions have many good properties. For instance, if a sequence of convex functions converge pointwise in a dense subset of an open subset U, then they converge in the whole U and the convergence is uniform on each compact subset of U. In this talk we will propose various notions of convexity on balanced quasi-embedded polyhedral spaces (to be defined during the talk) and show that they share the strong continuity properties of classical convex functions. We will also discuss the existence of Monge Ampere type measures for such functions.As an application of the theory we show that any semipositive toroidal b-divisor on an algebraic variety is integrable.This is join work in progress with A. Botero and M. Sombra.

Antoine Ducros: Flattening in Berkovich geometry

In a famous paper, Raynaud and Gruson have developped flattening techniques for coherent sheaves in scheme theory. I will present analogous results in the Berkovich setting and explain what kind of problems have to be overcome to adapt Raynaud and Gruson's methods in this realm.

Hannah Markwig: Modifications, faithful tropicalization and moduli spaces of plane tropical curves

Tropical geometry can be viewed as an efficient degeneration technique in algebraic geometry, with important applications for instance in enumerative geometry. To make good use of it, it can be necessary to focus on faithful tropicalizations. Modifications provide a concrete tool to construct faithful tropicalizations. We discuss the use and effect of modifications in the case of curves of genus one, two and three. This talk is based on joint work with Maria Angelica Cueto and with Marvin Hahn, Yue Ren and Ilya Tyomkin.

Johannes Nicaise: Motivic specialization and rationality problems

I will discuss an ongoing project with John Christian Ottem to find new examples of stably irrational hypersurfaces by combining the specialization results obtained with Evgeny Shinder with tropical compactification techniques. The guiding example will be the quartic fivefold.

Sam Payne: Topology of moduli spaces of tropical curves

I will discuss the topology of moduli spaces of stable tropical curves of volume 1, and present results on rational homology, from joint work with Chan, Faber, and Galatius, and some recent first steps toward understanding integral homology, from joint work with Allcock and Corey, in which we also show that these spaces are simply connected.

Joseph Rabinoff: Abelian schemes are log-Néron models

We will use non-Archimedean geometry in the style of Berkovich and Bosch–Ltkebohmert to prove that abelian schemes satisfy a Nron mapping property with respect to log smooth schemes.

Dhruv Raganathan: Three stories about curves in genus one

In the early days of tropical geometry, Speyer identified a beautiful and subtle combinatorial condition that distinguished tropical elliptic space curves from arbitrary balanced genus one graphs. Just before this, Vakil and Zinger had given a very explicit desingularization of the moduli space of elliptic curves in projective space, with remarkable applications. Just after this, Smyth constructed new compactifications of moduli spaces of pointed elliptic curves, using worse-than-nodal singularities, as part of the Hasset-Keel program. A decade on, we understand these three results as part of a single story involving logarithmic structures, their tropicalizations, and elliptic singularities. I will discuss this picture and how the unified framework extends all three results, and leads to new calculations in Gromov-Witten theory. These are joint projects with Luca Battistella, Yoav Len, Navid Nabijou, Keli Santos-Parker, and Jonathan Wise.

Johannes Rau: The dimension of amoebas

Amoebas $A(X)$ are images of algebraic varieties X in logarithmic coordinates and were introduced by Gelfand, Kapranov, Zelevinsky in their study of discriminants. From a "tropical" point of view, they appear as intermediate objects during the process of passing from the classical algebraic geometry to the piece-wise linear, combinatorial world of tropical geometry. However, basic properties of amoebas, even their dimensions, are not well-understood. In my talk, I will review some results and present a new formula computing $\dim(A(X))$, settling a conjecture by Nisse and Sottile. As a corollary, this formula implies that the amoeba dimension only depends on the tropicalization/Bergman fan of X. This is joint work with Jan Draisma et Chi Ho Yuen.

Michael Temkin: Reduction and lifting problems on Berkovich curves

The classical theory based on stable reduction theorem associates to any nice non-archimedean curve X over an algebraically closed non-archimedean field K a metric subgraph called skeleton and a reduction curve over the residue field k. These objects are unique up to easily described modifications, and in fact they are the only invariants of tropical and k-algebraic nature one may associate to X.
One may wonder if an analogous reduction theory exists for more involved objects, such as morphisms of curves $f : Y \to X$ or a curve X with a differential form w. It turns out that a straightforward approach based on simultaneous semistable reduction theorem or a naive reduction of w is in general not satisfactory (though it works fine when f is tame). This happens because there exist additional tropical invariants, such as the different and residue functions.
In this talk, I will describe these combinatorial invariants, introduce associated reduction over k, and will explain how lifting theorems indicate that the new invariants form a complete set in the following two cases:
(1) a wild $f : Y \to X$ with local degrees not divisible by p^2 (j/w U. Brezner),
(2) a pair (X, w) when the residue filed k is of characteristic 0 (j/w. I. Tyomkin)

Ilya Tyomkin: Irreducibility problem for Severi varieties

Severi varieties parameterize reduced irreducible curves of given geometric genus in a given linear system on an algebraic surface. The first irreducibility result for Severi varieties was established in 1986 and is due to Harris, who considered the classical case of planar curves in characteristic zero. Few more irreducibility results have been obtained since then but none of the known approaches is applicable in positive characteristic. In my talk Ill present the state of the art in the irreducibility problem, and discuss a new tropical approach which works also in the case of positive characteristic. The talk is based on a joint work with Karl Christ and Xiang He.

Martin Ulirsch: Tropicalizing the moduli space of curves with level structure

Due to the presence of automorphisms the moduli space of algebraic curves does not admit a fine moduli space as an algebraic variety. A classical way to avoid to inconvenience is to rigidify the moduli problem by introducing an extra topological datum on the algebraic curve, a so-called level structure. In this talk I will discuss several possible tropical analogues of level structures. The crucial notion of this story will the fundamental group of certain graph of groups (in the sense of Bass and Serre). Moreover, I will also discuss how to tropicalize the moduli space of algebraic curves with level structures both from a non-Archimedean and from a logarithmic point of view. This procedure depends on the structure of the tempered fundamental group (in the non-Archimedean case) and the logarithmic fundamental group (in the logarithmic case).
This talk is based on ongoing work with both Yoav Len and Dmitry Zakharov as well as with Mattia Talpo.

Annette Werner: Degenerations of vector bundles and Mustafin varieties

We look at degenerations of p-adic plane curves defined as the closure of the curve in a Mustafin variety, i.e a model of the projective plane which is given by a finite configuration of lattices in 3-space.
We exhibit a large class of such configurations which give rise to star-shaped curve degenerations. Then we investigate degenerations of syzygy bundles on plane curves. This is motivated by open questions in the framework of the p-adic Simpson correspondence. In particular, we answer a question by Holger Brenner about potentially strongly semistable reduction of a certain syzygy bundle on the Fermat curve.

Kazuhiko Yamaki: Ample divisors on tropical varieties

The notion of ample divisors on tropical curves is well known and has been studied by many authors. In this talk, we propose a definition of closed immersions (or faithful embeddings) and a notion of ample divisors on tropical varieties even of higher dimension. This is a joint work in progress with Shu Kawaguchi.

Tony Yue Yu: The Frobenius structure theorem for affine log Calabi-Yau varieties containing a torus

We show that the naive counts of rational curves in any affine log Calabi-Yau variety U, containing an open algebraic torus, determine in a surprisingly simple way, a family of log Calabi-Yau varieties, as the spectrum of a commutative associative algebra equipped with a compatible multilinear form. This is directly inspired by a very similar conjecture of Gross-Hacking-Keel in mirror symmetry, known as the Frobenius structure conjecture. Although the statement involves only elementary algebraic geometry, our proof employs Berkovich non-archimedean analytic methods. We construct the structure constants of the algebra via counting non-archimedean analytic disks in the analytification of U. We establish various properties of the counting, notably deformation invariance, symmetry, gluing formula and convexity. In the special case when U is a Fock-Goncharov skew-symmetric X-cluster variety, we prove that our algebra generalizes, and in particular gives a direct geometric construction of, the mirror algebra of Gross-Hacking-Keel-Kontsevich. It is joint work with S. Keel; the reference is arXiv:1908.09861.