Valter Moretti

An operational construction of the sum of two non-commuting observables in quantum theory and related constructions

The existence of a real linear-space structure on the set of observables of a quantum system i.e., the requirement that the linear combination of two generally non-commuting observables A, B is an observable as well is a fundamental postulate of the quantum theory yet before introducing any structure of algebra. However, it is by no means clear how to choose the measuring instrument of the composed observable $aA + bB (a, b \in R)$ if such measuring instruments are given for the addends observables A and B when they are incompatible observables. A mathematical version of this dilemma is how to construct the spectral measure of $f(aA+bB)$ out of the spectral measures of A and B. We present such a construction with a formula which is valid for generally unbounded selfadjoint operators A and B, whose spectral measures may not commute, and a wide class of functions $f : R \to C$. We prove that, in the bounded case the Jordan product of A and B can be constructed with the same procedure out of the spectral measures of A and B. The formula turns out to have an interesting operational interpretation and, in particular cases, a nice interplay with the theory of Feynman path integration and the Feynman-Kac formula.
Based on arXiv:1909.10974, joint work with N. Drago and S. Mazzucchi.