Workshop on PDEs for Surfaces and Interfaces

organised by

DFG-Research Unit 3013 *Vector- and Tensor-Valued Surface PDEs*
and DFG-Research Training Group 2339 *IntComSin*

Regensburg, 7–9 March 2022

Programme

Partial differential equations defined on surfaces play an important role in mathematics and in applications in the natural and engineering sciences. The workshop focuses on PDEs describing the evolution of an interface, on vector- and tensor-valued surface PDEs, and on PDEs on surfaces that are coupled to bulk equations. The workshop addresses modelling, analysis as well as numerics of PDEs for surfaces and interfaces.

Talks will take place in the lecture hall **H31**, while coffee breaks will be in the seminar room **M103**.

	Monday	Tuesday	Wednesday
9:00 – 9:45		Zhao	Bachini
9:45 – 10:15		Wittwer	Schwering
10:15 – 10:45	*Coffee break*		
10:45 – 11:30		Metzger	Bartels
11:30 – 12:00		Bonati	Doss
12:00 – 14:00	*Lunch break*		
14:00 – 14:45	Voigt	Padilla-Garza	
14:45 – 15:30	Salvalaglio	Praetorius	
15:30 – 16:00	*Coffee break*		
16:00 – 16:45	Garcke	Blank	
16:45 – 17:30	Kovács	Krause	
19:00		*Workshop dinner*	

Further informations

In the second half of the same week there is a workshop on **Mathematical Methods for Complex Phenomena**, jointly organised by the SPP 2256 and GRK 2339.

The workshop dinner will be held at the restaurant of Hotel Bischofshof. Address: Krauterermarkt 3, 93047 Regensburg.

Although, the workshop will be held in person, the talks can be followed online, via the zoom link https://uni-regensburg.zoom.us/j/68315898705?pwd=MTlZL05EK21YRUs3UlJQeEs2bzlldz09, Meeting ID: 683 1589 8705, Passcode: 562171.

Detailed programme

Dr. Elena Bachini (TU Dresden)

Tuesday, 14:00–14:45

Title: **An intrinsic finite element method for PDEs on surfaces**

Abstract: Surface PDEs have attracted the interest of many researcher over the last twenty years, due to their applications in various fields from fluid flow to biomedical engineering and electromagnetism. Many proposed numerical approaches rely on an embedding of the surface in a higher dimensional space. We present here an alternative finite element approach based on a geometrically intrinsic formulation, that we call Intrinsic Surface Finite Element Method (ISFEM). By careful definition of the geometry and the differential operators, we are able to arrive at an approximation that is fully intrinsic to the surface. We consider first a scalar advection-diffusion-reaction equation defined on a surface. In this case, the numerical analysis of the scheme is also available, and we show numerical experiments that support theoretical results. Then, we extend the differential operators for the case of vector-valued PDEs. In this case the presented formulation allows the direct discretization of objects naturally defined in the tangent space, without the need of any additional projection. Finally, we extend ISFEM to consider moving surfaces via an intrinsic re-definition of the PDE that takes into account a time-dependent metric tensor. To evaluate our approach, we consider several steady and transient problems involving both diffusion and advection-dominated regimes and compare its performance to established finite element techniques.

Prof. Dr. Sören Bartels (University of Freiburg)

Wedneday, 10:45–11:30

Title: **Modeling and simulation of nonlinear bending problems**

Abstract: The mathematical description of large bending deformations of thin elastic rods and plates leads to fourth order problems with nonlinear pointwise constraints that give rise to various free boundary phenomena. The free boundary may separate regions of trivial and large deformations, describe the contact zone in the presence of an obstacle, or be related to the occurrence of self-contact. We devise and analyse numerical methods that are capable of reliably capturing these effects under minimal regularity assumptions and which allow us to experimentally study topological transitions

Prof. Dr. Luise Blank (University of Regensburg)

Tuesday, 16:00–16:45

Title: **Optimal control of anisotropic Allen–Cahn equations**

Abstract: Anisotropic Allen-Cahn equations model for example the interface evolution in crystal growth. The focus of this talk is the control of such an evolution. Due to the anisotropy nondifferentiable terms occur which have to be treated appropriately to obtain an efficient optimization solver. Here we propose a particular regularization. The issue of differentiability also leads us to choose the implicit time discretization dG(0) where in addition energy stability is obtained.

First the existence of the control-to-state operator and its Lipschitz-continuity is shown for the time discretized as well as for the time continuous problem. Latter is based on the convergence proof of the discretized solutions. The existence of a global minimizer of the original and of the regularized problem is shown in the time discretized as well as in the continuous setting. Also the convergence with respect to regularization and to discretization is considered. Furthermore the Fréchet differentiability of the regularized problem is studied and optimality conditions are obtained.

Subsequently the trust-region Newton Steihaug-cg method is applied to the time discretized problem which is then discretized in space. This provides us with iteration numbers independent of the discretization level, where evidence is given numerically. Finally numerical examples with various anisotropies and configurations are presented.

The talk is based on a joint work with Johannes Meisinger (University of Regensburg).

Mirco Bonati, PhD cand. (TU Dresden)

Tuesday, 11:30–12:00

Title: **On the role of mechanosensitive binding dynamics in the pattern formation of active surfaces**

Abstract: Mechanochemical dynamics of active surfaces, as the thin cellular actin cortex, play a crucial role in several biological processes such as cell shape regulation and morphogenesis. Relying on a hydrodynamic theory of curved active surfaces, we study both theoretical and numerical aspects of the self-organized pattern formation of the cell cortex. In collaboration with Lucas D. Wittwer and Sebastian Aland (TUBF, FOR3013), we develop a model that takes into account biologically relevant facts, such as load-dependence of molecular unbinding and cortical stress stiffening. In particular, we study the influence of catch and slip bond cross-linkers on active gel pattern formation as it has been shown that the mechanical stiffness of the actin cytoskeleton can vary greatly with small changes in cross-linkers concentration. We find that this force-sensing gives rise to new aspects of pattern formation.

Martin Doss, PhD cand. (FAU Erlangen)

Wednesday, 11:30–12:00

Title: **Numerical study of single droplet drying in an acoustic levitator**

Abstract: We present a sharp interface model for the drying of an acoustically levitated droplet containing water and suspended solids. More precisely, a standing sound wave is used to immobilize the droplet in one of its pressure nodes. Once a critical amount of water has evaporated from the droplet, its liquid suspension turns into a porous medium consisting of a wet core and a dry crust. The drying rate is directly derived from the thermodynamic non-equilibrium between the saturated and the actual vapor pressure at the free phase boundary. Numerical simulations are performed to investigate the aforementioned phase transitions and to quantify the impact of the acoustic levitation on the single droplet drying kinetics.

Prof. Dr. Harald Garcke (University of Regensburg)

Monday, 16:00–16:45

Title: **On the interaction of mean curvature flow and diffusion on evolving hypersurfaces**

Abstract: We consider a geometric problem consisting of an evolution equation for a closed hypersurface coupled to a parabolic equation on this evolving surface. More precisely, the evolution of the hypersurface is determined by a scaled mean curvature flow that depends on a quantity defined on the surface via a diffusion equation. This system arises as a gradient flow of a simple energy functional. Assuming suitable parabolicity conditions, we derive short-time existence for the system. Afterwards, several properties of the solution are analyzed. Emphasis is placed on to what extent the surface in our setting evolves the same as for the usual mean curvature flow. To this end, we show that the surface area is strictly decreasing but give an example of a surface that exists for infinite times nevertheless. Moreover, mean convexity is conserved whereas convexity is not. We also construct an embedded hypersurface that develops a self-intersection in the course of time.

Finally, we discuss how solutions can be computed numerically with the help of an evolving surface finite element discretization. We will discuss optimal error bounds and present numerical experiments illustrating the above discussed qualitative properties of the flow as well as the convergence behaviour.

The analytical part is a joint work with Helmut Abels and Felicitas Bürger (both University of Regensburg) and the numerical part is joint work with Charlie Elliott (University of Warwick) and Balázs Kovács (University of Regensburg).

Dr. Kovács Balázs (University of Regensburg)

Monday, 16:45–17:30

Title: **Higher-order linearly implicit full discretization of the Landau–Lifshitz–Gilbert equation**

Abstract: In this talk we will explore the convergence and stability results for the Landau–Lifshitz–Gilbert (LLG) equation of micromagnetics. We will discuss linearly implicit backward difference formula (BDF) time discretizations up to order 5 combined with higher-order non-conforming finite element space discretizations, which are based on the weak formulation due to Alouges but use approximate tangent spaces that are defined by averaged rather than pointwise orthogonality constraints. We prove stability and optimal-order error bounds in the situation of a sufficiently regular solution.

The main focus of the talk will lie on the energy estimates for the stability analysis of the BDF time discretisation. We will highlight the key idea for the energy estimates using a continuous perturbation result, and track this approach through the different temporal and spatial discretisations.

The talk is based on joint work with Georgios Akrivis (Ioannina), Michael Feischl (Vienna), and Christian Lubich (Tübingen).

Veit Krause, PhD cand. (TU Dresden)

Monday, 16:45–17:15

Title: **A numerical approach for fluid deformable surfaces with incompressible enclosed volume**

Abstract: We model fluid deformable surfaces by the surface Navier–Stokes model under the assumption of elastic properties of the surface and incompressibility of the enclosed volume. We combine the model with an approach for moving hypersurfaces and discuss the discretization by surface finite elements (SFEM) on higher order curved triangulation. The model behavior will be shown in different simulations.

Dr. Stefan Metzger (FAU Erlangen)

Tuesday, 10:45–11:30

Title: **On the numerical treatment of Cahn-Hilliard equations with rate dependent dynamic boundary conditions**

Abstract: Important qualitative features of two-phase systems related to phase separation processes can be described by Cahn-Hilliard equations. For these equations, many different boundary conditions are available. While the simplest boundary conditions dictate a static contact angle and prevent flux across the boundary, more sophisticated models use additional partial differential equations to describe effects like dynamic contact angles or mass transfer across the boundary. Recently, a family of models using Cahn-Hilliard type equations on the boundary of the domain to describe adsorption processes was analyzed (cf. Knopf, Lam, Liu, M., M2AN, 2021). This family includes the case of instantaneous adsorption processes studied by Goldstein, Miranville, and Schimperna (Physica D, 2011) as well as the case of vanishing adsorption rates which was investigated by Liu and Wu (Arch. Ration. Mech. Anal., 2019). This talk addresses the numerical treatment of these models. We will present a fully discrete, unconditionally stable, linear finite element scheme based on the scalar auxiliary variable technique and discuss its convergence properties.

Dr. David Padilla-Garza (TU Dresden)

Tuesday, 14:00 – 14:45

Title: **A homogenized bending theory for prestrained plates**

Abstract: In this talk, we derive an effective bending plate model via simultaneous homogenization and dimension reduction. Our starting point is a 3D nonlinear elasticity model describing a composite whose components are prestrained with a magnitude that scales with the thickness of the plate. We assume that both the composite as well as the prestrain feature a periodic microstructure. After deriving the effective model via Gamma-convergence, we specialize in a class of examples that are explicitly solvable using a combination of analysis and numerics. Within this class of examples, we find several interesting and counterintuitive phenomena.

Dr. Simon Praetorius (TU Dresden)

Tuesday, 14:45–15:30

Title: **Tangential Errors of Tensor Surface Finite Elements**

Abstract: We discretize a tangential tensor field equation using a surface-finite element approach with a penalization term to ensure almost tangentiality. It is natural to measure the quality of such a discretization intrinsically, i.e., to examine the tangential convergence behaviour in contrast to the normal behaviour. We show optimal order convergence with respect to the tangential quantities in particular for an isogeometric penalization term that is based only on the geometric information of the discrete surface.

Dr. Marco Salvalaglio (TU Dresden)

Monday, 14:45–15:30

Title: **Mesoscale modeling of defects and interfaces in crystals by the amplitude phase-field crystal model**

Abstract: The Phase-Field Crystal (PFC) model describes atoms in a lattice through a continuous periodic density field and investigates diffusive time scales. In the amplitude expansion of the PFC model (APFC), a coarse-grained description of this density is obtained by focusing on its complex amplitudes and, in turn, on their dynamics. These amplitudes vary on length scales larger than the atomic spacing but still retain details of the crystal lattice. After outlining the basics of this approach, numerical simulations based on the APFC model and the Finite Element Method are shown to reproduce defects structures and interfaces in two and three dimensions for different crystal symmetries. The description of deformation fields derived from the complex amplitudes, their connections with the elasticity theory, and the characterisation of dislocations are then discussed. Finally, representative applications are illustrated, outlining near future perspectives.

Paul Schwering, PhD cand. (RWTH Aachen)

Wednesday, 9:45–10:15

Title: **On derivations of evolving surface Navier-Stokes equations**

Abstract: In recent literature several derivations of incompressible Navier-Stokes type equations that model the dynamics of an evolving fluidic surface have been presented. These derivations differ in the physical principles used in the modeling approach and in the coordinate systems in which the resulting equations are represented. This paper has overview character in the sense that we put five different derivations of surface Navier-Stokes equations into one framework. This then allows a systematic comparison of the resulting surface Navier-Stokes equations and shows that some, but not all, of the resulting models are the same. Furthermore, based on a natural splitting approach in tangential and normal components of the velocity we show that all five derivations that we consider yield the same tangential surface Navier-Stokes equations.

Prof. Dr. Axel Voigt (TU Dresden)

Monday, 14:00–14:45

Title: **Multiphase field models to describe epithelia tissue**

Abstract: We consider a modelling approach which describes each cell in an epithelia tissue by a phase field variable. Cell-cell interactions are considered by an interaction potential and subcellular details responsible for motility are resolved on different levels. We describe the modelling and the numerical approach and analyse simulation results with respect to mechanical properties, emerging positional and orientational order, topological defects and emerging dynamics. We are especially interested in the influence of the considered subcellular details and these macroscopic properties and compare with experimental results.

Lucas Wittwer, PhD cand. (TU Bergakademie Freiberg)

Tuesday, 9:45–10:15

Title: **A computational model of self-organized shape dynamics of active surfaces in fluids**

Abstract: Mechanochemical processes play a crucial role during morphogenesis, the formation of complex shapes and tissues out of a single cell. On the cellular level, the actomyosin cortex governs shape and shape changes. This thin layer of active material underneath the cell surface exerts an active contractile tension, the strength of which being controlled by the concentration of force-generating molecules. Advective transport of such molecules leads to a complex interplay of hydrodynamics and molecule concentration which gives rise to pattern formation and self-organized shape dynamics like cell division. In this talk, we present a novel numerical model to simulate such active surfaces immersed in fluids. We show the resulting patterning and cell shape dynamics for different parameter configurations away from the linear regime. Together with Mirco Bonati and Elisabeth Fischer-Friedrich (FOR3013, Physics of Life) we extend the analytical model to consider biological relevant behaviour of the actomyosin cortex and investigate the resulting dynamics and pattern formation.

Dr. Quan Zhao (University of Regensburg, Humboldt-fellow)

Tuesday, 9:00–9:45

Title: **Structure-preserving parametric finite element methods for surface diffusion**

Abstract: In this talk, I will present structure-preserving parametric finite element methods (SP-PFEM) for discretizing the surface diffusion of a closed curve in two dimensions (2D), a surface in three dimensions (3D). Here the "structure-preserving" refers to preserving the two fundamental geometric structures of the surface diffusion flow: (i) the conservation of the area/volume enclosed by the closed curve/surface, and (ii) the decrease of the perimeter/total surface area of the curve/surface. The proposed schemes are based on weak formulations that allow tangential degrees of freedom, where the exact area/volume conservation is maintained with the help of suitably weighted discrete normals. The methods are implicit and the resulting polynomial nonlinear system can be solved via Newton's method. The generalization of the SP-FEM to the axisymmetric surface diffusion, the anisotropic surface diffusion for surfaces attached to external plane boundaries will also be considered.