

## Title and abstracts

- Klaus Kröncke (Universität Hamburg )

### **Stability of ALE Ricci-flat manifolds under Ricci flow**

We prove that if an ALE Ricci-flat manifold  $(M, g)$  is linearly stable and integrable, it is dynamically stable under Ricci flow, i.e. any Ricci flow starting close to  $g$  exists for all time and converges modulo diffeomorphism to an ALE Ricci-flat metric close to  $g$ . By adapting Tian's approach in the closed case, we show that integrability holds for ALE Calabi-Yau manifolds which implies that they are dynamically stable.

This is joint work with Alix Deruelle.

- Tadano Homare (Tokyo University of Science)

### **Some Cheeger-Gromov-Taylor Type Compactness Theorems for Ricci Solitons**

An important problem in Riemannian geometry is to investigate the relation between topology and geometric structure on Riemannian manifolds. The celebrated theorem of S. B. Myers [6] guarantees the compactness of a complete Riemannian manifold under some positive lower bound on the Ricci curvature. This theorem may be considered as a topological obstruction for a complete Riemannian manifold to have a positive lower bound on the Ricci curvature. On the other hand, J. Lohkamp [5] proved that in dimension at least three, any manifold admits a complete Riemannian metric of negative Ricci curvature. Hence, in dimension at least three, there are no topological obstructions to the existence of a complete Riemannian metric of negative Ricci curvature. To give an interesting compactness criterion for complete Riemannian manifolds is one of the most important problems in Riemannian geometry, and the Myers theorem has been widely generalized in various directions by many authors. The aim of this talk is to discuss the compactness of complete Ricci solitons. Ricci solitons were introduced by R. Hamilton in 1982 and are natural generalizations of Einstein manifolds. They correspond to self-similar solutions to the Ricci flow and often arise as singularity models of the flow. The importance of Ricci solitons was demonstrated by G. Perelman, where Ricci solitons played crucial roles in his affirmative resolution of the Poincaré conjecture. In this talk, after we reviewed basic facts on Ricci solitons, we shall establish some new compactness theorems for complete Ricci solitons [11]. Our results may be regarded as natural generalizations of the compactness theorem due to J. Cheeger, M. Gromov, and M. Taylor [1] and improve previous compactness theorems obtained by M. Fernández-López and E. García-Río [2], M. Limoncu [3, 4], Z. Qian [7], the author [8, 9, 10], and G. Wei and W. Wylie [12].

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- [8] H. Tadano, *Remark on a diameter bound for complete Riemannian manifolds with positive BakryEmery Ricci curvature*, Diff. Geom. Appl. 44 (2016), 136143.
- [9] H. Tadano, *An upper diameter bound for compact Ricci solitons with application to the HitchinThorpe inequality*, J. Math. Phys. 58 (2017), 023503, 8 pages.
- [10] H. Tadano, *Some Ambrose- and Galloway-type theorems via BakryEmery and modified Ricci curvatures*, to appear in Pacific J. Math. (2018).
- [11] H. Tadano, *Some CheegerGromovTaylor type compactness theorems via  $m$  BakryEmery and  $m$ modified Ricci curvatures*, Preprint (2016).
- [12] G. Wei and W. Wylie, *Comparison geometry for the BakryEmery Ricci tensor*, J. Differential Geom. 83 (2009), 377405.

- Volker Branding (Universität Wien)

#### **The heat flow for the full bosonic string**

The full bosonic string action is an important energy functional in modern theoretical physics. It couples the Dirichlet energy for a map between a surface and a manifold to a scalar and a two-form potential. In this talk we will study the heat flow associated to the full bosonic string action and present several existence results for its critical points.

- Felix Lubbe (Universität Hamburg)

#### **Mean curvature flow of maps between non-compact manifolds**

Given a smooth map  $f$  between two Riemannian manifolds  $M$  and  $N$  satisfying certain curvature assumptions, the mean curvature flow of graph of  $f$  can be used to induce a homotopy on the map. In this talk, I will discuss the situation of  $M$  and  $N$  being Euclidean spaces and  $f$  satisfying a length-decreasing condition. In this case, the flow has a smooth long-time solution consisting of graphs of length-decreasing maps. Further, I will give an outlook on the case where  $N$  is a complete manifold with sectional curvature bounded from above by a negative constant.

- Friederike Dittberner (Universität Konstanz)

#### **Area-preserving curve shortening flow**

This talk is about the enclosed area preserving curve shortening flow for non-convex embedded curves in the plane. I will show that initial curves with a lower bound of  $-\pi$  on the local total curvature stay embedded under the flow and develop no singularities in finite time. Moreover, the curves become convex in finite time and converge exponentially and smoothly to a round circle.

- Mattia Fogagnolo (University of Trento)

#### **A Willmore inequality on ALE manifolds**

The classical Willmore inequality states that the  $L^2$  norm of the mean curvature of any smooth surface embedded into the Euclidean space is bounded from below by  $16\pi$  and the equality is achieved if and only if the surface is a round sphere. Using new monotonicity formulas, we show how to get a corresponding inequality on every Asymptotically Locally Euclidean (ALE) manifolds with nonnegative Ricci curvature. The result applies to the case of gravitational instantons.

Joint work with V. Agostiniani and L. Mazziere.

- Marco Pozzetta (University of Pisa)

#### **Confined Willmore energy and the Area functional**

In this talk we will recall the basic definition and properties of the Willmore energy as seen as a functional from a set of surfaces to the real numbers. We will discuss the concept of varifold and the related approach to the variational problem for the Willmore functional. Finally we will present a new class of functionals  $W$  defined as the difference between the Willmore energy of a surface and its area with the latter multiplied by a constant positive weight. We will give results of existence and partial regularity for a minimization problem for such functionals using the varifold approach described before.

- Julia Menzel (Universität Regensburg)

#### **Willmore flow of planar networks**

We consider networks of curves in the plane moving according to the  $L^2$ -gradient flow of a variant of the elastic energy. In this talk I will discuss short time existence in the case of networks composed by three curves that are required to meet in one or two triple junctions. As a variation of the result we additionally impose that they form an angle of 120 degrees at the triple junction(s).

The presented result is joint work with Harald Garcke and Alessandra Pluda.

- Shokhrukh Kholmatov (University of Vienna)

#### **Minimizing Movements for mean curvature flow of partitions**

In this talk I would like to discuss about minimizing movements solutions of mean curvature flow of bounded partitions of space.

This is a joint work with G. Bellettini.

- Adrian Spener (Universität Ulm)

#### **The elastic flow of curves in hyperbolic space**

In this talk we study the one-dimensional analogue of the Willmore flow of closed curves in the hyperbolic space. We show well-posedness and long time existence of the flow, and prove sub-convergence under additional length-penalisation. We briefly discuss the necessity of the assumption of penalisation. If time permits we consider the elastic flow in the sphere and discuss similar results

Joint work with Anna Dall'Acqua.

- Afuni Ahmad (FU Berlin)

**Singularities in and local regularity of the harmonic map and Yang-Mills heat flows**

We shall discuss a new approach to studying the singularities occurring in second-order geometric flows, particularly the harmonic map and Yang-Mills heat flows on complete Riemannian manifolds. The techniques here are centred around new monotonicity and energy identities for these flows built on suitable 'heat balls.'

- Lothar Schiemanowski (Universität Kiel)

**A blow up criterium for the spinor flow on surfaces**

The spinorial energy functional is a functional on the space of metrics whose critical points are special holonomy metrics in dimension 3 and higher. The spinor flow is its gradient flow. On surfaces the functional has a different geometric interpretation. I will report on recent work concerning the formation of singularities. I will show how to derive a blow up criterium based on a compactness theorem for metrics on surfaces and the decomposition of the flow into a conformal evolution and a movement of constant curvature metrics.

- Stephen Lynch (FU Berlin)

**Pinched ancient solutions to the high codimension mean curvature flow**

Huisken and Sinestrari have demonstrated that the only compact, uniformly convex ancient solution to the mean curvature flow of hypersurfaces is the homothetically shrinking sphere. We will prove an analogue of their theorem for solutions of arbitrary codimension which, in place of uniform convexity, makes use of a quadratic pinching condition on the second fundamental form. Andrews and Baker have shown that solutions satisfying this pinching condition at the initial time vanish in round points, and we will see how our results lead to a new proof of their theorem. A key ingredient in all of this is the observation that pinching of the second fundamental form forces positivity of the full curvature operator. This is joint work with Huy Nguyen.

- Sebastian Hensel (Institute of Science and Technology Austria)

**Weak-strong uniqueness for Navier-Stokes two-phase flow with surface tension**

We establish a weak-strong uniqueness principle for the flow of two immiscible, incompressible and viscous fluids with surface tension under the assumption of identical viscosities. As long as there exists a strong solution to the system, every varifold solution originating from the same initial condition has to coincide with it. Our result covers the regime of phase-dependent densities and holds true in two and three spatial dimensions. The global-in-time existence of varifold solutions was established by H. Abels (Interfaces Free Bound. 9, 2007). The key ingredient of our result is the construction of a relative entropy functional which is capable of controlling the interface error.

- Giuseppe Pipoli (Università degli Studi dell'Aquila)

**Inverse mean curvature flow in complex hyperbolic space**

We consider the evolution by inverse mean curvature flow of a closed, mean convex and star-shaped hypersurface in the complex hyperbolic space. We prove that the flow is defined for any positive time, the evolving hypersurface stays star-shaped and mean convex. Moreover the induced metric converges, after rescaling, to a conformal multiple of the standard sub-Riemannian metric on the sphere. Finally we show that there exists a family of examples such that the Webster curvature of this sub-Riemannian limit is not constant.

- Julia Butz (Universität Regensburg)

### **Curve Diffusion Flow with a Contact Angle**

We consider the evolution of a curve due to the curve diffusion in two dimensions. This geometric evolution equation arises in problems of phase separation in material science and is the two-dimensional version of the surface diffusion flow. We consider the flow subject to boundary conditions that prescribe the position of the end point to be on a fixed line and a fixed angle with that line. First we will discuss a result on well-posedness locally in time for initial curves of low regularity. Then we will discuss how this result can be used to obtain a blow-up criterion in term of on  $L^2$ -bound of the curvature.

This is a joint work with Helmut Abels.

- Julian Scheuer (Universität Freiburg)

### **Locally constrained inverse curvature flows**

Let  $N^{n+1} = (R_0, \infty)S^n$  be a Riemannian warped product with metric

$$\bar{g} = dr^2 + \lambda(r)^2\sigma,$$

where  $\sigma$  is the round metric on  $S^n$  and  $\lambda$  is a warping function. We consider graphical (over  $S^n$ ) time-dependent embeddings  $x : (0, T) \times M^n \rightarrow N^{n+1}$  of a closed manifold  $M^n$  moving according to *curvature* flows of the form

$$\dot{x} = -f(\mathcal{W}, u, r)\nu \tag{0.1}$$

where  $f$  depends on the Weingarten operator  $\mathcal{W}$  of the hypersurface  $M_t = x(t, M)$ , the support function  $u = \langle \lambda \partial_r, \nu \rangle$  and the radial direction  $r$  and where  $\nu$  is the outward normal vector field along  $M_t$ . The function  $f$  can be arranged to preserve geometric quantities. The prototype speed with this property was first studied by Guan/Li [1] with

$$f = H u n \lambda(r), \tag{0.2}$$

where  $H$  is the mean curvature. This flow preserves the enclosed volume, while decreasing the surface area, making it a natural candidate to prove the isoperimetric problem in such warped spaces. Fully nonlinear versions of this flow (preserving other quermassintegrals) have been treated in the Euclidean space [2]. These flows have the big advantage that they do not contain a global term (hence we call them *locally constrained flows*), which is usually added in order to keep geometric quantities fixed, but forces strong restrictions onto the initial hypersurfaces, such as convexity, in order to obtain convergence results. Note that in order to deal with the speed (0.2), only the

starshapedness of the flow hypersurfaces was assumed. Hence it is desirable to deduce convergence results for fully nonlinear versions of (0.2) and apply them to obtain geometric inequalities in general ambient spaces. In this talk we present a recent result in this direction, cf. [3], namely we obtain long-time existence and  $C^\infty$ -convergence to a slice  $r = \text{const}$  for the flow (0.1) with the speed

$$f = \frac{u}{\lambda} \frac{n}{F},$$

where  $F$  is a 1-homogeneous curvature function (such as the mean curvature) satisfying few properties. We apply this result to obtain new Minkowski-type inequalities for starshaped hypersurfaces in the AdS-Schwarzschild manifolds and in the hyperbolic space.

Joint work with Chao Xia.

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