Book of Abstracts for the Workshop
"Analysis of Fluid and Elastic Bodies Interactions"
April 11-13, 2022

organized by Helmut Abels, Yadong Liu, Maria Neuss-Radu

Schedule

Time	Monday	Tuesday	Wednesday
9:00-9.45		Hillairet	Muha
9:45-10:15		Trautwein	Breit
10:15-10:45		*coffee break*	*coffee break*
10:45-11:15		Knoch	Češík
11:15-12:00		Gahn	Schwarzacher
12:00-14:00		*lunch break*	
14:00-14:45	Tucsnak	Kukavica	
14:45-15:15	Ghosh	Krier	
15:15-15:45	*coffee break*	*coffee break*	
15:45-16:15	Valášek	Liu	
16:15-16:45	Su	Roy	
16:45-17:30	Nečasová	Čanić	
19:00		*Workshop dinner*	

Abstracts (in Order of Talks)

Long time behavior for the system describing the motion of a rigid in a viscous fluid

Marius Tucsnak (University of Bordeaux)

We discuss the system describing the motion of a rigid body in a viscous incompressible fluid. It is assumed that the fluid-rigid system fills the whole 3-dimensional space. The focus is on the large time behavior of solutions. We first investigate the case of a spherical rigid body and then we discuss the case of a solid of arbitrary shape.

Thin film model of a meniscus formation

Amrita Ghosh (HCM University of Bonn)

I would like to discuss a different mathematical model describing the sinking of a solid wedge into a Newtonian liquid, forming meniscus, in thin film setting. We obtain a thin film equation, similar to the classical one. Interestingly, the no-slip boundary condition under this particular geometry gives rise to a non-degenerate fourth order equation which is well-posed, unlike the classical one. Existence of a strong solution can be shown, at least for short time, using standard result for quasilinear parabolic problem. At the end, I will discuss briefly the steady state for such thin film model which shows different behavior compared to the classical model.

The influence of different inlet boundary conditions in FE approximation of fluid-structure interaction problem

Jan Valášek (Czech Technical University in Prague)

The contribution deals with the numerical simulation of fluid-structure interaction problem, here represented by the human vocal folds vibration excited by the fluid flow. The main attention is paid to the inlet boundary conditions. The classical Dirichlet boundary condition in the form of prescribed velocity has the drawback of unphysical pressure increase during channel closing phase at each vocal folds vibration cycle. The another often used possibility is to prescribe pressure drop between inlet and outlet by the the do-nothing type of boundary condition. lt usually leads to quite high oscillation of inlet velocity. In order to overcome these disadvantages, the penalization approach is investigated,where beside the given pressure drop the inlet velocity is weakly enforced with the aid of the penalization term. This is quite original approach within scope of finite element method, although usual approach within discontinuous Galerkin methods. The vocal folds are modelled as an elastic isotropic body with assumption of small displacements. Due to the small velocities compared to the speed of sound the fluid flow can be described by the incompressible Navier-Stokes equations. For the purpose of numerical simulation of the time varying computational domain the arbitrary Lagrangian-Euler method is applied. The whole problem is solved by the finite element method based solver. Numerical results will be presented and analyzed.

Control problem of a floating body system in shallow water

Pei Su (Charles University Prague)

We consider the control problem describing the interactions of water waves with a rigid body partially immersed in a bounded container. The body is allowed to move only in the vertical direction. The fluid is modeled by the shallow water equations. The control signal is a vertical force acting on the floating body. The system above respresents a class of wave energy converters called the *"point absorber device"* in engineering. We first derive the full governing equations of the fluid-body system in a water tank and reformulate them as an initial boundary value problem of a first-order evolution system, only defined in a part of the bottom of the domain in the horizontal direction. Then we linearize the equations around the equilibrium state and study its well-posedness. Finally, we focus on the description of the reachable space and stabilizability of the linear system. Our main result asserts that, provided that the floating body is situated in the middle of the tank, any symmetric waves with appropriate regularity can be obtained from the equilibrium state by an appropriate control force. It implies that we can project this system on the subspace of states with symmetry properties to obtain a reduced system which is approximately controllable and strongly stabilizable. We also obtain an explicit non-uniform decay rate in this case. In general, this system is not controllable (even approximately). Moreover, we discuss the special case when the object floats at one lateral boundary, which is somewhat different in the nonlinear setting. This is a joint work with M. Tucsnak (Bordeaux).

Existence of a weak solution to a nonlinear fluid-structure interaction problem with heat exchange

Šárka Nečasová (Czech Academy of Sciences)

We study a nonlinear interaction problem between a thermoelastic shell and a heat-conducting fluid. The shell is governed by linear thermoelasticity equations and encompasses a time-dependent domain which is filled with a fluid governed by the full Navier–Stokes–Fourier system. The fluid and the shell are fully coupled, giving rise to a novel nonlinear moving boundary fluid-structure interaction problem involving heat exchange. The existence of a weak solution is obtained by combining three approximation techniques – decoupling, penalization and domain extension. In particular, the penalization and the domain extension allow us to use the methods already developed for compressible fluids on moving domains. In such a way, the proof is more elegant and the analysis is drastically simplified. Let us stress that this is the first time the heat exchange in the context of fluid-structure interaction problems is considered. It is a joint work with Václav Mácha, Boris Muha, Arnab Roy and Srdjan Trifunović.

Modelling compressible bubbly flows

Matthieu Hillairet (Université de Montpellier)

When a viscous fluid carries a dispersed phase (solid particles or gas bubbles), the properties of the dispersed phase may modify in return the rheology of the leading fluid. A classical issue is then to compute the properties of the mixture seen as a plain fluid. In this talk, I will focus on the compressible case when the dispersed phase is also made of fluid. In this case, formal method for deriving a multiphase model are proposed in classical monographs (see for instance "Theory of multicomponent fluids" by D.A. Drew and S.L. Passman). This talk is devoted to possible rigorous analytical justifications of these formal approaches. It is based on joint work with D. Bresch and with H. Mathis and N. Seguin.

Viscoelastic Cahn–Hilliard models for tumour growth

Dennis Trautwein (University of Regensburg)

We introduce a new phase field model for tumour growth where viscoelastic effects are taken into account. The model is derived from basic thermodynamical principles and consists of a convected Cahn–Hilliard equation with source terms for the tumour cells and a convected reaction diffusion equation with boundary supply for the nutrient. Chemotactic terms, which are essential for the invasive behaviour of tumours, are taken into account. The model is completed by a viscoelastic system constisting of the Navier–Stokes equation for the hydrodynamic quantities, and a general constitutive equation with stress relaxation for the left Cauchy–Green tensor associated with the elastic part of the total mechanical response of the viscoelastic material.

For a specific choice of the elastic energy density and with an additional dissipative term accounting for stress diffusion, we prove existence of global-in-time weak solutions of the viscoelastic model for tumour growth in two space dimensions $d = 2$ by the passage to the limit in a fully-discrete finite element scheme where a CFL condition, i.e. $\Delta t \leq Ch^2$ is required.

Moreover, in arbitrary dimensions $d \in \{2, 3\}$, we show stability and existence of solutions for the fully-discrete finite element scheme, where positive definiteness of the discrete Cauchy–Green tensor is proved with a regularization technique that was first introduced by Barrett and Boyaval. After that, we improve the regularity results in arbitrary dimensions $d \in \{2, 3\}$ and in two dimensions $d = 2$, where, for the latter, a CFL condition is required. Then, in two dimensions $d = 2$, we pass to the limit in the discretization parameters and show that subsequences of discrete solutions converge to a global-in-time weak solution.

Finally, we present numerical results in two dimensions $d = 2$.

Modelling and Simulation of Transport Processes in an Elastically Deformable Perforated Medium

Jonas Knoch (FAU Erlangen-Nuremberg)

We present in this talk an effective model for transport processes in periodically perforated elastic media, taking into account also cyclic elastic deformation as it occurs e.g. in lung tissue due to respiratory movement. The underlying microscopic problem consists of a linear elasticity equation for the displacement within the Lagrangian framework, posed on a fixed domain and a diffusion equation for the concentration within the Eulerian framework, posed on the current deformed domain. After a transformation of the diffusion equation onto the fixed domain, we derive the upscaled model by means of a formal asymptotic expansion. The system is nonlinearly coupled through effective coefficients, which also take into account the periodic microstructure. We develop and study numerical methods for our problem and perform simulations that are inspired by a bioengineered microdevice which is able to reconstitute critical lung functions (Lung-On-A-Chip). The simulations shed light into the sensitivity of the model with respect to several experimental parameters such as frequency or magnitude of the cyclic mechanical strain. This is a joint work with Markus Gahn (Heidelberg), Nicolas Neuß (Erlangen) and Maria Neuss-Radu (Erlangen).

Derivation of coupled Stokes-Plate-Equations for fluid flow through a thin porous elastic layer

Markus Gahn (University Heidelberg)

We consider fluid flow through a thin porous elastic layer, which separates two fluid-filled bulk domains. The thin periodically perforated layer consists of a fluid and an elastic solid part. Thickness and periodicity of the layer are of order ϵ, where the parameter ϵ is small compared to the size of the bulk domains. The evolution of the fluid flow is described by an instationary Stokes system, and the solid via linear elasticity. We derive a macroscopic model with effective interface laws using rigorous homogenization and dimension reduction methods for $\epsilon \to 0$, when the thin layer reduces to an interface Σ separating the two bulk domains. To pass to the limit $\epsilon \to 0$ we use multiscale techniques adapted to problems in continuum mechanics, including extension operators for perforated domains preserving ϵ-uniform bounds for the symmetric gradient, Korn-inequalities, and two-scale compactness of ϵ-dependent sets in Sobolev spaces on thin perforated layers. We show that the effective model consists of the Stokes equations in the bulk domains coupled to a time dependent plate equation on the interface Σ including homogenized elasticity coefficients carrying information about the micro structure of the layer. The macroscopic fluid velocity is continuous at the interface, where only a vertical movement occurs and the tangential components vanish. The macroscopic displacement is given as a Kirchhoff-Love displacement. We further identify higher order correctors for the fluid velocity and for the displacement in the thin layer.

The global existence for a fluid-structure interaction system

lgor Kukavica (University of Southern California)

We address a system of partial differential equations modeling a motion of an elastic body inside an incompressible fluid. The fluid is modeled by the incompressible Navier-Stokes equations while the structure is represented by the wave equation. We will review known local and global existence theorems. The most recent global existence result is joint with A. Tuffaha and W. Ozanski.

Hieracrchic Asymptotically Based Algorithm for Simulation of Viscous Fluid Througha Thin Fibrous Layer

Maxime Krier (ITWM Fraunhofer Kaiserslautern) joint work with Julia Orlik, Grigory Panasenko

Stokes fluid is flowing through a spacer fabric, a three dimensional textile structure which can be modeled by a porous layer between two parallel hyperplanes with periodically distributed parallel beam lattices, which are orthogonal to the hyperplanes. The flow direction is parallel to the hyperplanes and orthogonal to the lattices. The thickness of the lattices or porous layers is a small parameter ε. The fluid viscosity is assumed to be of order $\varepsilon^3 E$, where E is the Young's modulus of the beams.

Fluid-solid interaction is considered in the structure; the deflection of the beam lattices due to pressure jumps in the fluid is of interest for the practical application. A dimension reduction as the lattice thickness $\varepsilon \to 0$ is presented in [1]. The lattice-layers are replaced by their homogenized mean surfaces leading to Stokes flow with a non-standard interface condition: the pressure jump at the mean surfaces is proportional to the biharmonic operator in the surfaces applied to the velocity trace. In the limit, the normal component of the macroscopic velocity field is an H^2-function of the lattice mean-plane variable and the limit problem is non-local in time. This corresponds to the non-stationarity of the initial problem.

The dimension reduction approach enables a decoupling of the numerical solving process into two main steps, drastically reducing computational effort compared to the initial fluid-solid interaction problem with the fully resolved lattice structures (see [2],[3],[4]). The approach results in a linear integro-differential equation which is reformulated to an equivalent stiff linearly implicit ODE with a total of 12 degrees of freedom per mesh node, representing fluid velocity and interface displacements in three dimensions.

Convergence estimates from the corresponding analysis will be used to estimate the numerical accuracy of the reduced dimension algorithm. Finally, local stresses in the beams and the fluid pressure will be reconstructed as in [2] with the help of interpolated and extended piece-wise polynomial function sequences which are strongly convergent to the solution.

References

[1] J. Orlik, G. Panasenko and R. Stavre, *Asymptotic analysis of a viscous fluid layer separated by a thin stiff stratified elastic plate*. Applicable Analysis, **100**(3), 589–629, 2021.

[2] G. Griso, L. Khilkova, J. Orlik and O. Sivak, *Asymptotic Behavior of Stable Structures Made of Beams*. Journal of Elasticity, **143**, 239–-299, 2021.

[3] J. Orlik, G. Panasenko and V. Shiryaev, *Optimization of textile-like materials via homogenization and dimension reduction.* SIAM Multiscale Model. Simul., **14**(2), 637–667, 2016.

[4] G. Griso, A. Migunova, J. Orlik, *Asymptotic Analysis for Domains Separated by a Thin Layer Made of Periodic Vertical Beams.* J. Elast., **128**(2), 291–331, 2017.

Short time existence of a quasi-stationary fluid-structure interaction problem for plaque growth

Yadong Liu (University of Regensburg)

We address a quasi-stationary fluid-structure interaction problem coupled with cells reaction and growth, which comes from the plaque formation during the stage of the atherosclerotic lesion in human arteries. The blood is modeled by the incompressible Navier-Stokes equation, while the motion of vessels is captured by the equilibrium of an incompressible nonlinear hyperelastic equation. The growth happens when both cells in fluid and solid react, diffuse and transport across the interface, resulting in the accumulation of foam cells, which are exactly seen as the plaques. Via a fixed-point argument, we derive the local well-posedness of the nonlinear system, which is sustained by the analysis of decoupled linear systems. This is a joint work with Helmut Abels (Uni. Regensburg).

Analysis of 1D Blood flow in a network of vessels

Arnab Roy (BCAM Bilbao, Spain)

We talk about the well-posedness of a system of hyperbolic/parabolic model regarding blood flows in a network of vessels with viscoelastic walls. We prove the existence and uniqueness of maximal strong solution. We also discuss stability estimates under suitable nonlinear Robin boundary conditions.

Some recent results on a deterministic, multi-layered, fluid-poroelastic structure interaction problem, and a stochastic fluid-structure interaction problem, motivated by biological applications

Sunčica Čanić (University of California, Berkeley)

This talk has two parts, each addressing certain aspects of fluid-structure interaction problems. In the first part we present a complex, multi-scale model, and a recent well-posedness result in the area of fluid-poroelastic structure interaction, which have helped the design of a first implantable bioartificial pancreas without the need for immunosuppressant therapy. We show global existence of a weak solution to a fluid-structure interaction (FSI) problem between the flow of an incompressible, viscous fluid, modeled by the time-dependent Stokes equations, and a multi-layered poroelastic medium consisting of a thin poroelastic plate and a thick poroelastic medium modeled by a Biot model. This is the first global (weak) solution existence result in the context of poroelastic FSI. Numerical simulations of the underlying problem showing optimal design of a bioartificial pancreas, will be presented. This is a joint work with bioengineer Shuvo Roy (UCSF), and mathematicians Yifan Wang (UCI), Lorena Bociu (NCSU), Boris Muha (University of Zagreb), and Justin Webster (University of Maryland, Baltimore County). In the second part of this talk we present a stochastic fluid-structure interaction problem describing the interaction between the flow of an incompressible viscous fluid and a linearly elastic membrane which is stochastically forced by white noise. The motivation derives from the stochastically forced coronary arteries moving on the surface of the heart which contracts and relaxed during the systolic and diastolic parts of the cardiac cycle. We discuss the existence of a weak solution in the probabilistically strong sense for this stochastic PDE problem, and emphasize its special scaling properties that allow the solution to exist. This is joint work with PhD student Jeffrey Kuan at UC Berkeley.

Fluid - Poroelastic Structure Interactions

Boris Muha (University of Zagreb)

We consider the interaction between an incompressible, viscous fluid modeled by the dynamic Stokes equation and a multilayered poroelastic structure which consists of a thin, linear, poroelastic plate layer (in direct contact with the free Stokes flow) and a thick Biot layer. The fluid flow and the elastodynamics of the multilayered poroelastic structure are fully coupled across a fixed interface through physical coupling conditions (including the Beavers-Joseph-Saffman condition), which present mathematical challenges related to the regularity of associated velocity traces. We prove existence of weak solutions to this fluid-structure interaction problem with either (i) a linear, dynamic Biot model, or (ii) a nonlinear quasi-static Biot component, where the permeability is a nonlinear function of the fluid content (as motivated by biological applications). The proof is based on constructing approximate solutions through Rothe's method, and using energy methods and a version of Aubin-Lions compactness lemma (in the nonlinear case) to recover the weak solution as the limit of approximate subsequences. We also provide uniqueness result for the linear problem and a weak-strong uniqueness type of result for the nonlinear problem.

Finally, we discuss some further direction of research such as diffuse interface methods and moving boundary fluid - poroelastic structure interaction problems.

The presented results are joint work with L. Bociu, M. Bukač, S. Čanić and J. Webster.

Navier-Stokes-Fourier fluid interacting with elastic shells

Dominic Breit (Heriot-Watt University Edinburgh)

We study the motion of a compressible heat-conducting fluid in three dimensions interacting with a nonlinear flexible shell. The fluid is described by the full Navier-Stokes Fourier system. The shell constitutes an unknown part of the boundary of the physical domain of the fluid and is changing in time. The solid is described as an elastic non-linear shell of Koiter type; in particular it possesses a non-convex elastic energy. We show the existence of a weak solution to the corresponding system of PDEs which exists until the moving boundary approaches a self-intersection or the non-linear elastic energy of the shell degenerates. Our solutions comply with the first and second law of thermodynamics: the total energy is preserved, and the entropy balance is understood as a variational inequality. This is a joint work with S. Schwarzacher.

Energy estimates in a variational approach to hyperbolic evolutions

Antonín Češík (Charles University, Prague)

In a recent work, B. Benefova, M. Kampschulte and S. Schwarzacher introduced a variational time-stepping method for solving (non-linear) hyperbolic problems with viscoelastic solids, possibly interacting with a fluid. lt is an extension of the famous minimizing movements principle, using two time scales: the velocity scale and the (larger) acceleration scale. In their method, they first pass with the velocity scale to 0, to obtain energy estimates which then allow to pass with the acceleration scale to 0. In this talk, we dive deeper into the energy estimates to see how to allow for a simultaneous limit passage of the two parameters to zero. This is a joint work with Sebastian Schwarzacher.

On the construction of weak solutions describing elastic bulk solids interacting with fluids

Sebastian Schwarzacher (Charles University, Prague)

In this talk some recent existence results in fluid-structure interactions are discussed. lt focuses on the case when bulk solids, which potentially deform largely are interacting with fluids. One of the characteristic difficulties of the respective PDE systems is the variable-in-time fluid domain being a part of the solution. The construction of solutions is by step-wise minimization. Such a variational approximation seems to be irreplaceable for large deformation solids, since the respective state spaces are (for physical reasons) non-convex. We introduce a two time-scale approximation scheme that is capable to construct second-order in time PDEs via step-wise minimization. Further it allows to construct weak solutions describing bulk solids interacting with fluids governed by the incompressible or compressible Navier-Stokes equations. The talk is based on collaborations with B. Benesova, D. Breit and M. Kampschulte.