1 ADF Test Simulation

The four following tables were produced by a simulation with two slightly different DGPs

$$y_t = 0.6y_{t-1} + 0.3y_{t-2} + u_t, \quad u_t \sim N(0,5) \tag{1}$$

$$y_t = 0.6y_{t-1} + 0.4y_{t-2} + u_t, \quad u_t \sim N(0,5) \tag{2}$$

where the first (D1) is stationary with roots $z_{11} = 1.08, z_{12} = -3.08$ and the second (D2) contains a unit root, i. e. $z_{21} = 1, z_{22} = 2.5$. For each of these DGPs, there were two simulations with sample sizes $T_1 = 200$ and $T_2 = 50$, respecetively. The other parameters were fixed, i. e. for all simulations the DGPs were replicated 10.000 times, a significance level of 0.05 was applied, all three types of ADF tests were applied (Type - 0: no intercept, no trend; Type - μ: intercept, no trend; Type-τ: intercept, trend) and $p = 0, \ldots, 10$ additional lags were included.

The following four tables show the percentage of rejections (to be more precise: the number of cases when the test statistic was smaller then the 5% critical value divided by 10.000) depending on the type applied and the number of additional lags included (augmented). The correct number of lags to include is 2.

	D1	$T_1 =$	200	D1	$T_2 = 50$	50	D2	$T_1 =$	200	D2	$T_2 =$	50
p	0	μ	τ	0	μ	τ	0	μ	τ	0	μ	τ
0	1.00	0.98	0.94	0.59	0.39	0.37	0.23	0.28	0.42	0.23	0.25	0.36
1	0.97	0.60	0.38	0.26	0.09	0.07	0.05	0.05	0.05	0.05	0.05	0.05
2	**0.95**	**0.56**	**0.34**	**0.23**	**0.08**	**0.06**	**0.05**	**0.05**	**0.05**	**0.05**	**0.04**	**0.04**
3	0.93	0.53	0.32	0.22	0.07	0.06	0.05	0.05	0.05	0.05	0.04	0.05
4	0.91	0.49	0.29	0.19	0.06	0.05	0.05	0.05	0.05	0.04	0.04	0.04
5	0.89	0.46	0.27	0.18	0.06	0.05	0.05	0.05	0.05	0.05	0.04	0.04
6	0.87	0.43	0.24	0.16	0.05	0.04	0.04	0.05	0.05	0.04	0.04	0.04
7	0.85	0.40	0.22	0.15	0.05	0.05	0.05	0.05	0.04	0.04	0.04	0.04
8	0.82	0.37	0.21	0.14	0.05	0.04	0.04	0.05	0.04	0.04	0.03	0.03
9	0.80	0.35	0.19	0.13	0.05	0.04	0.04	0.05	0.04	0.04	0.04	0.04
10	0.77	0.32	0.17	0.11	0.04	0.04	0.04	0.04	0.04	0.04	0.03	0.03

Before going into the interpretation, remember that the ADF test has the containment of a unit root under the null and remember the statistical measures for test outcomes

	DGP is H_0 (H_0 is true)	DGP is not H_0 (H_0 is false)
Test points to H_1	false positive (type-I)	true negative (sensitivity / power)
Test points to H_0	true positive (specificity)	false negative (type-II)

and realize that inclusion of 2 lags is optimal for the test design. This means that D1 corresponds to true H_1 while D2 corresponds to true H_0.
Interpretation:

- D1-200: With increasing lags, the percentage of rejections decreases. By correctly specifing the test (type-0 and p = 2), in 95% of the cases the test decides correctly. In the case of the other test types, one looses rejections.
- D1-50: In small samples (time series of length 50) the number of rejections is quite low although it is a stationary time series. Even in the correct specification scenario, one has only 23% rejections.
- D2-200/D2-50: By defining $\alpha = 0.05$, we restrict type-I error to be bounded by α. This is the case here. Only in 5% of all test applications the test points to a stationary time series although it contains a unit root (except in the case of not including lags at all for which there are high rejection rates).

Schriftliche Versicherung

Ich versichere, dass die Arbeit von mir selbständig verfasst wurde und dass ich keine anderen als die angegebenen Quellen und Hilfsmittel verwendet habe. Weiterhin wurde diese Arbeit keiner anderen Prüfungsbehörde übergeben.

Regensburg, den 14. Juni 2016